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The measurement of the electronic g-factor in hydrogen-like ions —A promising tool for determining fundamental and nuclear constants

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Abstract. We describe a double–Penning-trap experiment suitable for testing QED in strong fields by determining the electronic g-factor of a single hydrogen-like ion in its ground state. Our measurements on $^{12}C^{5+}$ reach a relative accuracy of 2×10^{-9}, where the largest uncertainty results from the mass of the electron. Together with equally precise theoretical predictions therefore, it is possible to evaluate a new value for the electron's mass. The possibilities to obtain other fundamental constants and nuclear parameters are lined out.

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1 Introduction

Quantum electrodynamics can claim to be the most precise theory of microscopic physics. The g-factor of the free electron, *e.g.*, was measured to be $g = 2(1 + a)$ with $a_{\mathrm{exp}} = 1\,159\,652\,188\,4(43) \times 10^{-13}$ [1]. The prediction by theory is $a = 1\,159\,652\,216\,0(12)(678) \times 10^{-13}$ [2], where the first error is due to insufficiently known higher-order terms and the second is due to the value of the fine-structure constant α employed in the evaluation. In turn, the $g - 2$ experiment at present allows the most precise determination of α.

The success of QED for light particles is impressive. However, the theory is less well studied for very strong electric and magnetic fields. At present, considerable effort is undertaken to enlarge our knowledge about QED into that region. Highly charged ions still provide the highest accessible field strengths [3,4]. For the Lamb shift, experiments are performed up to hydrogen-like uranium [5].

We have developed and tested a setup for studying g-factors of hydrogen-like systems with a precision of up to a few times 10^{-10} [6,7]. Other charge states of single ions are also planned to be investigated. In the present contribution we will briefly describe our experiment and results, compare them with the theoretical predictions and outline some possibilities to determine fundamental and nuclear constants by our atomic-physics setup.

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2 Experimental

A single hydrogen-like ion is stored in the magnetic field (3.8 T) of a Penning trap. A g-factor measurement takes place by determining the Larmor precession frequency

$$\omega_{\mathrm{L}} = g \frac{e}{2\,m_{\mathrm{e}}} B, \tag{1}$$

where e is the positive fundamental charge unit and m_{e} is the mass of the electron. By g we denote the g-factor of the system electron + spin-zero nucleus for a suitable hydrogen-like ion. In (1) B is the magnetic-field strength which is connected to the cyclotron frequency of the stored ion by

$$\omega_{\mathrm{c}}^{\mathrm{i}} = \frac{q_{\mathrm{i}}}{m_{\mathrm{i}}} B, \tag{2}$$

where q_{i} and m_{i} denote charge and mass of the ion, respectively. Therefore g can be obtained from

$$g = 2 \frac{\omega_{\mathrm{L}}}{\omega_{\mathrm{c}}^{\mathrm{i}}} \frac{q_{\mathrm{i}}}{e} \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}} \tag{3}$$

and only the frequency ratio $\omega_{\mathrm{L}}/\omega_{\mathrm{c}}^{\mathrm{i}}$ has to be measured. This is performed by irraditating the ion by microwaves of the frequency ω_{mw} and investigating the number of induced spin-flips as a function of the frequency ratio $\omega_{\mathrm{mw}}/\omega_{\mathrm{c}}^{\mathrm{i}}$. We measure $\omega_{\mathrm{c}}^{\mathrm{i}}$ by observing the image currents induced in the trap electrode by the orbiting ion, simultaneously to the irradiation. The magnetic field in this

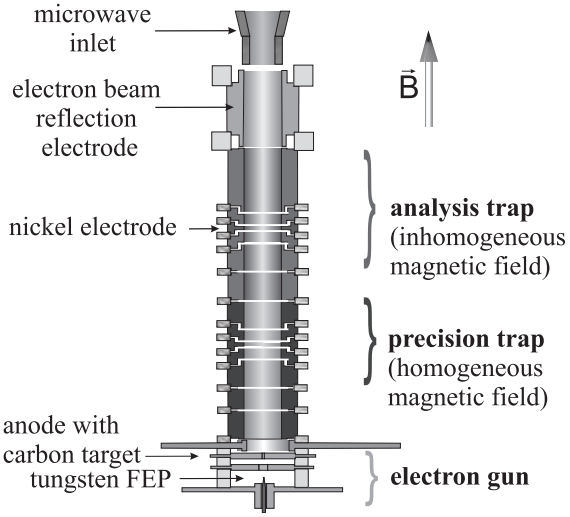

Fig. 1. Sketch of the traps.

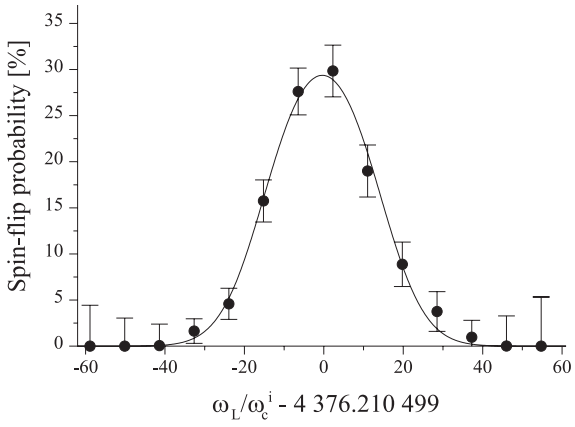

Fig. 2. Normalized Larmor resonance for $^{12}C^{5+}$ measured in the precision trap. Displayed is the spin-flip probability *vs.* the ratio ω_{mw}/ω_c. Along the x-axis, the values indicate the deviation from the center in units of 10^{-6}. The solid line is a fit of a Gaussian.

"precision trap" is rather homogeneous. The harmonic term of the expansion $B = B_0 + B_2\, z^2 + \ldots$ amounts to $B_2 = 8\ \mu\mathrm{T/mm}^2$.

To investigate whether a spin-flip actually has taken place, the ion is moved to a second adjacent trap ("analysis trap"), where the magnetic-field inhomogeneity is larger by three orders of magnitude ($B_2 = 10\ \mathrm{mT/mm}^2$). The inhomogeneity is generated by a nickel ring electrode. In this field, the frequency of the axial motion of the stored ion differs slightly for both possible spin orientations, and this difference can be measured [6,8]. After determining the spin state, the ion is moved back to the precision trap and the process is repeated in order to determine the spin-flip rate as a function of the applied microwave frequency.

A scheme of our trap is shown in fig. 1. In total, it consists of a stack of 13 cylindrical electrodes of 0.7 cm inner diameter. The two potential minima (traps) can be created along the axis, separated by about 2 cm. Single highly charged ions are stored for months, and neither vacuum ($p < 10^{-16}$ mbar) nor transport between the two traps ever restricted the storage time of an ion up to now. In addition, no "uncontrolled" spin-flips were observed during the transport.

Sweeping over an interval of ω_{mw}, a resonance curve like the one shown in fig. 2 is obtained. The center of the curve can be determined within 1% of the relative linewidth which is 7×10^{-9} for the case shown. We correct for the residual magnetic inhomogeneity in the precision trap by extrapolating to vanishing oscillation amplitudes. This extrapolation and the unknown asymmetry of the lineshape currently limit our accuracy. A more detailed description of the experiment and the evaluation techniques is provided in [9,10].

3 Results

Up to now, we have investigated $g(^{12}C^{5+})$. For this quantity, the current theoretical prediction is given by

$$g(^{12}C^{5+}) = 2.001\,041\,589\,9\ (26). \tag{4}$$

The error margin results from numerical uncertainties and approximations. On a level of 10^{-11}, also the present uncertainty in the published value of the fine-structure constant $\alpha = 1/137.035\,999\,76(50)$ [11] affects the theoretical prediction. It enters via the analytically given Dirac value [12]

$$g_{\,\mathrm{Dirac}} = \frac{2}{3}\left(1 + 2\sqrt{1-(Z\alpha)^2}\right) \tag{5}$$

of the g-factor for the ground state of a hydrogen-like ion with nuclear charge Z. A detailed listing of all theoretical contributions is presented in table 1.

The value and also the table deviate slightly from that quoted in [7] because here we have employed the new CODATA value for α in our calculations. In addition, the existing $Z\alpha$ expansion for QED corrections of the order

Table 1. Theoretical contributions to $g(^{12}C^{5+})$. Except for "QED, bd., $(\alpha/\pi)^2$" all numerical values are taken from [4]. The rows "Dirac theory" and "QED, free, order (α/π)" were adjusted for the new CODATA value for α [11]. Where no error is given, it is less than one unit of the last digit. The error for the bound-state QED of order (α/π) is purely numerical, the errors for recoil and bound-state QED, order $(\alpha/\pi)^2$, result from employing perturbation series in $Z\alpha$. In order not to underestimate any systematic effect, the numerical errors are linearly added.

Dirac theory (incl. binding)	1.998 721 354 4
Finite-size correction	+0.000 000 000 4
Recoil (in $Z\alpha$ expansion)	+0.000 000 087 6 (9)
QED, free, order (α/π)	+0.002 322 819 5
QED, bound, order (α/π)	+0.000 000 844 2 (12)
QED, free, $(\alpha/\pi)^2$ to $(\alpha/\pi)^4$	−0.000 003 515 1
QED, bd., $(\alpha/\pi)^2$, $(Z\alpha)^2$ term	−0.000 000 001 1 (5)
Total theoretical value	2.001 041 589 9 (26)

$(\alpha/\pi)^2$ has been taken into account. In [7], a rather large error margin was employed instead. For that expansion, only the leading term is known [13–16], given by

$$g_{J\,(\alpha/\pi)^2,\,(Z\alpha)^2} = 2\,\left(\frac{\alpha}{\pi}\right)^2\,\frac{(Z\alpha)^2}{6}\,\times\,(-0.328\ldots), \quad (6)$$

where the last number is the numerical value of the corresponding term in the commonly employed expansion of the g-factor for the free electron, cf. formula (B6) in [11]. In order not to underestimate any higher-order terms in the corresponding $Z\alpha$ expansion, we assign an error margin of 50% to this value. In this way, also bound-state terms of order $(\alpha/\pi)^3$ and higher are thought to be taken into account. For a closer discussion of all other contributions, we refer to [4,17].

Experimentally, we obtain from the ratio $\omega_{\rm L}/\omega_{\rm c}^{\rm i}$ (cf. (3)) [7]

$$g_J(^{12}{\rm C}^{5+}) = 2.001\,041\,596\,3\,(10)(44), \quad (7)$$

where the first error is due to our experimental uncertainty and the second one results from the published value for the mass of the electron [11],

$$m_{\rm e} = 0.000\,548\,579\,911\,0\,(12)\ {\rm u}, \quad (8)$$

which is mainly based on the experiment of Farnham *et al.* [18]. In comparison, the mass of $^{12}{\rm C}^{5+}$ is well known, since neutral ^{12}C defines the atomic-mass unit and the consideration of five missing electrons and their binding energies only results in a relative uncertainty of 10^{-13}.

4 The mass of the electron

Our experiment forms one of the most stringent (and rather successful) tests of QED in any multicharged system up to now. It is almost self-evident to change the point of view and determine the most imprecise quantity from the comparison of experiment and theory; in other words, to obtain a new value for the mass of the electron. Such a value is largely independent of that of [18] and forms an important consistency check as explicitly recommended in [11]. By comparing (4) and (7), we obtain [19]

$$m_{\rm e} = 0.000\,548\,579\,909\,2\,(8)\ {\rm u}, \quad (9)$$

which agrees with (8) within its error margins. Employing the current CODATA value for the atomic mass of the proton, $m_{\rm p} = 1.00727646688(13)$u, we find for the proton-electron mass ratio

$$\frac{m_{\rm p}}{m_{\rm e}} = 1836.125\,673\,3\,(26)\ {\rm u}, \quad (10)$$

which can be compared to the published value,

$$\frac{m_{\rm p}}{m_{\rm e}} = 1836.125\,667\,5\,(39)\ {\rm u}. \quad (11)$$

It should be pointed out that the measurement procedure in [18] (and also in the preceding experiments [20–23]) was different from ours. There, ions and electrons were stored subsequently in a Penning trap, and their mass ratio was obtained by measuring their cyclotron frequencies. That procedure requires enormous care because the storage of electrons and positively charged ions requires a change in the trapping potential and thus both particle species may be located at different positions in the trap and experience a different magnetic field. Also the subsequent measurements of the cyclotron frequencies require a high temporal stability of the magnetic field [24]. On the other hand, our method allows to obtain the mass ratio from one single ion, and the crucial values of $\omega_{\rm L}$ and $\omega_{\rm c}^{\rm i}$ are determined simultaneously. A related experiment employing laser fluorescence was performed by Wineland *et al.* [25] investigating $g(^9{\rm Be}^+)$. However, in not very heavy lithium-like ions, the electron-electron interaction still forms an additional obstacle for very precise theoretical predictions even though recently considerable progress has been made ([26] and refs. therein).

5 Other basic values

We are not restricted to hydrogen-like ^{12}C. Any ion can be investigated, provided it is possible to bring it into our trap as proposed for the HITRAP facility at GSI [27]. This also offers several possibilities for determining other basic values some of which we are going to point out now.

Fine-structure constant. It was already mentioned that the theoretical prediction to the g-factor for $^{12}{\rm C}^{5+}$ is sensitive to the current error margin of α on the 10^{-11} level. From (5) it can be deduced that

$$\frac{\delta\alpha}{\alpha} \sim \frac{1}{(Z\alpha)^2}\frac{\delta g}{g}, \quad (12)$$

which means that a precise determination of α is possible from heavier systems. For light nuclei, (12) has to be treated with some care as the total (free + bound) QED effects of order (α/π) are of the same size but of opposite sign as the pure binding correction, given by $g_{\rm Dirac} - 2$ and therefore for carbon, *e.g.*, $\delta\alpha/\alpha \sim 10^{-3}$ at our current level of precision. With the same experimental and theoretical precision for calcium, however, $\delta\alpha/\alpha \sim 1.5 \times 10^{-8}$ seems feasible which is only one order of magnitude worse than that of the current CODATA value but of similar accuracy as the best non-QED determination of α [28] (cf. also [11]). For a determination of α from hydrogen-like uranium, the precision would even match the one from the g-factor of the free electron, provided both our experiment and the theoretical prediction are as precise as for carbon.

Nuclear radii. The effect of the nuclear size on the g-factor in carbon amounts to 4×10^{-10} (cf. table 1). For uranium, the nuclear-size effect amounts to 1.3×10^{-3} [17]. The uncertainty for this value imposed by the current error margin of the nuclear size, measured by Zumbro *et al.* [29], $\langle r^2\rangle^{1/2}(^{238}{\rm U}) = 5.8604(23)$ fm, amounts to 10^{-7}. From the theoretical side, it is not difficult to take into account an arbitrary distribution for the nuclear electric charge, provided it is well known. Therefore, also here the arguments can be turned around and a precise measurement of the g-factor serves as a sensible probe for the

nuclear size and shape. In addition, for well-known nuclear radii of one isotope of an element, the determination of the change in the rms radius between two isotopes can be performed even beyond the current knowledge of QED since the difference of the g-factors for these isotopes δg, can be expressed as a function of $Z\alpha$,

$$\delta g \simeq A(Z\alpha)\,\delta\langle r^2\rangle^{1/2}\,, \tag{13}$$

where the QED contribution cancels out because it is the same for both systems if they possess the same number of electrons.

Nuclear magnetic moments. A pending problem not only at the joint between nuclear and atomic physics is the precise knowledge of nuclear magnetic moments μ_I. Most of the tabulated values [30] are obtained either by NMR (nuclear magnetic resonance) measurements on ions in a solution or by atomic spectroscopy. In both cases, the measured value has to be corrected for the effect of the electronic cloud which is known as "diamagnetic-shielding correction". In addition, some of the measurements taking place in solutions seem to point to an environment-sensitive effect, termed chemical shift. The necessary corrections to obtain the "pure" magnetic moments have caused some inconsistencies in the literature. A recent overview about the problem is given by [31] which also contains further references. It is clear that a "cleaner" method to determine nuclear magnetic moments would be highly appreciable.

For ions with nuclear spin, the total g_F-factor is given by

$$g_F = g_J \frac{F(F+1)+J(J+1)-I(I+1)}{2F(F+1)} - \frac{m_\mathrm{e}}{m_\mathrm{p}} g_I \frac{F(F+1)+I(I+1)-J(J+1)}{2F(F+1)}, \tag{14}$$

where g_J is the electronic g-factor which was discussed so far, and g_I is the nuclear g-factor. Electronic, nuclear, and total angular momentum are denoted by J, I, and F, respectively. If g_F is measured with a precision of the order 10^{-9} and g_J is known at the same level from theory or from experiments on an isotope of the same element with $I = 0$, this still leaves a precision of 10^{-6} for g_I which is competitive to most of the tabulated values. In addition, no further corrections would have to be performed and diamagnetic shielding and chemical shift could be experimentally checked for the first time.

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References

1. R.S. Van Dyck jr., P.B. Schwinberg, H.G. Dehmelt, Phys. Rev. Lett. **59**, 26 (1987).
2. V.W. Hughes, T. Kinoshita, Rev. Mod. Phys. **71**, 133 (1999).
3. P.J. Mohr, G. Plunien, G. Soff, Phys. Rep. **293**, 227 (1998).
4. T. Beier, Phys. Rep. **339**, 79 (2000).
5. T. Stöhlker, P.H. Mokler, F. Bosch, R.W. Dunford, F. Franzke, O. Klepper, C. Kozhuharov, T. Ludziejewski, F. Nolden, H. Reich, P. Rymuza, Z. Stachura, M. Steck, P. Swiat, A. Warczak, Phys. Rev. Lett. **85**, 3109 (2000).
6. N. Hermanspahn, H. Häffner, H.-J. Kluge, W. Quint, S. Stahl, J. Verdú, G. Werth, Phys. Rev. Lett. **84**, 427 (2000).
7. H. Häffner, T. Beier, N. Hermanspahn, H.-J. Kluge, W. Quint, S. Stahl, J. Verdú, G. Werth, Phys. Rev. Lett. **85**, 5308 (2000).
8. R.S. Van Dyck jr., P.B. Schwinberg, H.G. Dehmelt, Phys. Rev. D **34**, 722 (1986).
9. M. Diederich, H. Häffner, N. Hermanspahn, M. Immel, H. Kluge, R. Ley, R. Mann, W. Quint, S. Stahl, G. Werth, Hyperfine Interact. **115**, 185 (1998).
10. H. Häffner, T. Beier, S. Djekic, N. Hermanspahn, H.-J. Kluge, W. Quint, S. Stahl, J. Verdú, T. Valenzuela, G. Werth, *Double Penning trap technique fro precise g factor determination in highly charged ions*, to be published in Eur. Phys. J. D.
11. P.J. Mohr, B.N. Taylor, Rev. Mod. Phys. **72**, 351 (2000).
12. G. Breit, Nature (London) **122**, 649 (1928).
13. H. Grotch, Phys. Rev. Lett. **24**, 39 (1970).
14. M.I. Eides, H. Grotch, Ann. Phys. (New York) **260**, 191 (1997).
15. S.G. Karshenboim, Phys. Lett. A **266**, 380 (2000).
16. A. Czarnecki, K. Melnikov, A. Yelkhovsky, Phys. Rev. A **63**, 012509 (2001).
17. T. Beier, I. Lindgren, H. Persson, S. Salomonson, P. Sunnergren, H. Häffner, N. Hermanspahn, Phys. Rev. A **62**, 032510 (2000).
18. D.L. Farnham, R.S. Van Dyck jr., P.B. Schwinberg, Phys. Rev. Lett. **75**, 3598 (1995).
19. T. Beier, H. Häffner, N. Hermanspahn, S.G. Karshenboim, H.-J. Kluge, W. Quint, S. Stahl, J. Verdú, G. Werth, Phys. Rev. Lett. **88**, 011603 (2002).
20. G. Gärtner, E. Klempt, Z. Phys. A **287**, 1 (1978).
21. G. Gräff, H. Kalinowsky, J. Traut, Z. Phys. A **297**, 35 (1980).
22. R.S. Van Dyck jr., P. B. Schwinberg, Phys. Rev. Lett. **47**, 395 (1981).
23. R.S. Van Dyck jr., F.L. Moore, D.L. Farnham, P.B. Schwinberg, Int. J. Mass Spectrom. Ion Proc. **66**, 327 (1985).
24. R.S. Van Dyck jr., D.L. Farnham, P.B. Schwinberg, Phys. Scr. T **59**, 134 (1995).
25. D.J. Wineland, J.J. Bollinger, W.M. Itano, Phys. Rev. Lett. **50**, 628; 1333 (1983).
26. Z.-C. Yan, Phys. Rev. Lett. **86**, 5683 (2001).
27. W. Quint, J. Dilling, S. Djekic, H. Häffner, N. Hermanspahn, H.-J. Kluge, G. Marx, R. Moore, D. Rodriguez, J. Schönfelder, G. Sikler, T. Valenzuela, J. Verdú, C. Weber, and G. Werth, Hyperfine Interact. **132**, 457 (2001).
28. A.-M. Jeffrey, R.E. Elmquist, L.H. Lee, R.F. Dziuba, IEEE Trans. Instrum. Meas. **46**, 264 (1997).
29. J.D. Zumbro, E.B. Shera, Y. Tanaka, C. E. Bemis jr., R.A. Naumann, M.V. Hoehn, W. Reuter, R.M. Steffen, Phys. Rev. Lett. **53**, 1888 (1984).
30. R.B. Firestone, *Table of Isotopes*, 8th edition (John Wiley & Sons, New York, 1996).
31. M.G.H. Gustavsson, A.-M. Mårtensson-Pendrill, Phys. Rev. A **58**, 3611 (1998).